Akerlof's Problem

In some markets buyers use a market statistic to judge the quality of prospective purchases. This provides an incentive for sellers to market poor quality since the returns to good quality accrue to the entire group of sellers rather than those with good quality. Thus there tends to be a reduction in average quality and the size of the market. Notice that the social and private returns differ in the market.

Automobile Market

This example provides an alternative explanation for the price difference between new and used cars, i.e., those that have left the showroom. For simplicity, suppose there are new and used cars and good and bad cars.

The consumer purchases a new car without knowing whether it is a good car or a lemon. However, consumers know it is good with probability θ and a lemon with probability $1 - \theta$.

After owning the car for awhile the owner is able to re-estimate the probability that it's good. The buyers of used cars cannot estimate this probability and so an asymmetry of information exists in the used car market. Since buyers cannot distinguish between good and bad cars, they must all sell at the same price. Clearly, used cars cannot have the same value as new cars since it would allow lemon owners to sell a used car and purchase a new car at a higher probability of being good. The new good car owner is locked in since he cannot receive the true value, or the expected value, of the car. Thus bad cars tend to drive out good cars.

Asymmetric Information

Akerlof considers an example of a market in which even worse pathologies exist, i.e., bad driving out not so bad driving out medium driving out not so good driving out good.

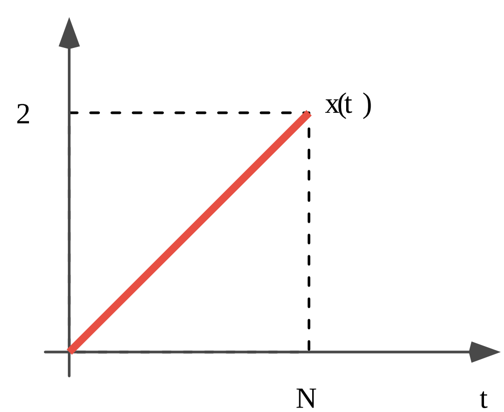
Suppose there are two groups of traders. Let the first group have N cars with uniformly distributed quality x such that $0 \le x \le 2$, and the utility function

$$u(y, n) = y + \int_n^N x(t)\, dt \tag{1}$$

where x(t) is the density of quality per unit of good two and y is the quantity of good one. Let

$$x(t) = \frac{2t}{N},\ t \in [0, N] \tag{2}$$

The density is shown in the next figure.



$$\int_{n}^{N} \frac{2t}{N}\, dt = \left.\frac{t^2}{N}\right|_{n}^{N} = N - \frac{n^2}{N} \tag{3}$$

Let N(x) denote the number of cars with quality less than or equal to x. Since quality is uniformly distributed over the N cars, it follows that

$$t = N(x) = \frac{1}{2} N x \tag{4}$$

For example, the number of cars with quality no more than one is one half of N. Then

$$x(t) = N^{-1}(x) = \frac{2t}{N} \tag{5}$$

Group one derives its income from car sales and so its problem is

$$\begin{aligned} &\text{maximize } y + \int_{n}^{N} x(t)\, dt \\ &\text{subject to } y = p\, n \end{aligned} \tag{6}$$

where p is the price of a car. Group two has no cars and its utility function is

$$V(y, n) = y + \int_{0}^{n} \frac{3}{2}\, x(t)\, dt \tag{7}$$

and its problem is

$$\text{maximize } E\left\{ y + \int_0^n \frac{3}{2} x(t)\,dt \right\} \qquad (8)$$
$$\text{subject to } y + p\,n = m$$

where m denotes income.

Group one supplies cars and knows the quality of each. Thus its supply may be determined by maximizing

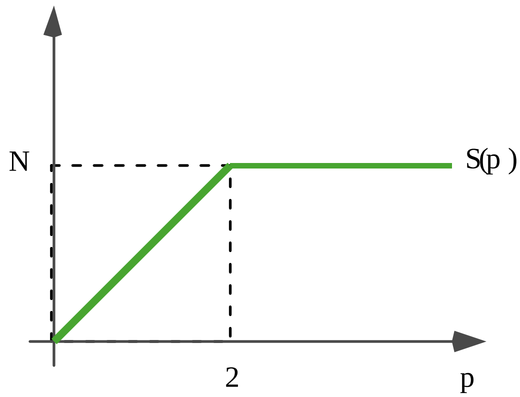
$$\begin{aligned} u(y, n) &= y + \int_n^N x(t)\,dt \\ &= y + N - \frac{n^2}{N} \end{aligned} \qquad (9)$$

Since y = p n, we obtain

$$p\,n + N - \frac{n^2}{N} \qquad (10)$$

which achieves a maximum when p = 2n/N, or equivalently n = p N/2. Therefore, the supply of cars is

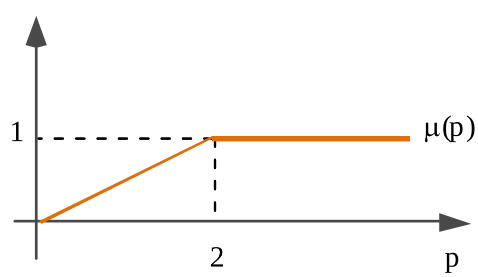
$$n = S(p) = \min\left\{ \frac{1}{2} p\,N, N \right\}$$



Next, let μ(p) denote the mean quality on the market at the price p. Intuitively, this is the total quality on the market at p divided by the total units on the market at p, i.e.,

$$\mu(p) = \frac{\int_0^{\frac{pN}{2}} x(t)\,dt}{\frac{pN}{2}} = \frac{1}{2}p \tag{11}$$

for p ≤ 2 and one otherwise.



Now, consider the buyer's maximization problem. Note that

$$E\left\{ y + \int_0^n \frac{3}{2} x(t)\,dt \right\} = \frac{3}{2}\ \mu\ n \tag{12}$$

It follows that the buyers solve the following constrained maximization problem

$$\begin{gathered} \max\text{imize } y + \frac{3}{2}\mu n \\ \text{subject to } y + pn = m \end{gathered}$$

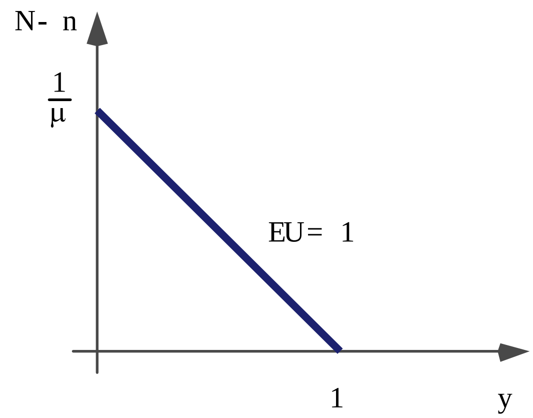
A representative indifference curve is shown in the next figure. The marginal rate of substitution is 2/(3 μ) while the price ratio is 1/p. It follows that the demand function is the standard form for perfect substitutes.[1] Therefore, demand is

[1] As with all perfect substitutes, the marginal rate of substitution is a constant. Hence, if the marginal rate of substitution exceeds the price ratio, equivalently, the marginal benefit of good one exceeds the marginal cost of that good in terms of good two then the agent purchases only good one and vice versa. The cases are as follows:

$$\text{(i)}\ \frac{1}{\frac{3}{2}\mu} > \frac{1}{p} \Leftrightarrow p > \frac{3}{2}\mu \Rightarrow n = 0$$

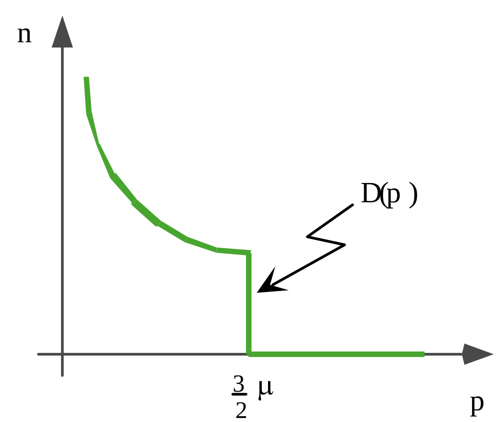
$$\text{(ii)}\ \frac{1}{\frac{3}{2}\mu} = \frac{1}{p} \Leftrightarrow p = \frac{3}{2}\mu \Rightarrow n \in \left[0, \frac{m}{p}\right]$$

$$\text{(iii)}\ \frac{1}{\frac{3}{2}\mu} < \frac{1}{p} \Leftrightarrow p < \frac{3}{2}\mu \Rightarrow n = \frac{m}{p}$$



$$n = D(p) = \begin{cases} 0 & p > \frac{3}{2}\mu \\ \left[0, \frac{m}{p}\right] & p = \frac{3}{2}\mu \\ \frac{m}{p} & p < \frac{3}{2}\mu \end{cases} \tag{13}$$

This demand is shown in the next figure.



However, since

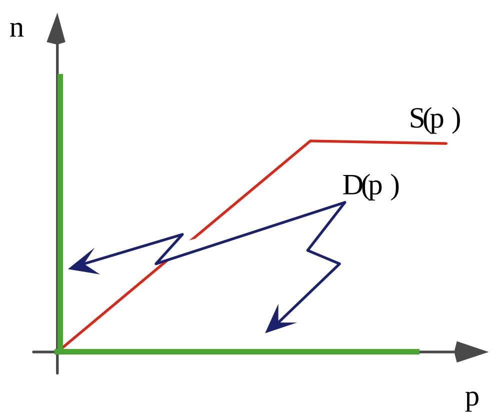
$$\mu(p) = \frac{1}{2}\, p < \frac{2}{3}\, p$$

for all p > 0 there exists no p > 0 such that $\mu(p) \geq \frac{2}{3}p$ and so demand is zero for all p > 0. Equivalently, observe that the marginal rate of substitution is greater than the price ratio for all positive prices since

$$\frac{1}{\frac{3}{2}\mu(p)} = \frac{1}{\frac{3}{4}p} > \frac{1}{p} \Leftrightarrow p > \frac{3}{4}p$$

It follows that automobile purchases are driven to zero as the agent uses all her income to purchase other goods. The following figure depicts Akerlof's example of market failure.

Figure: Market Failure

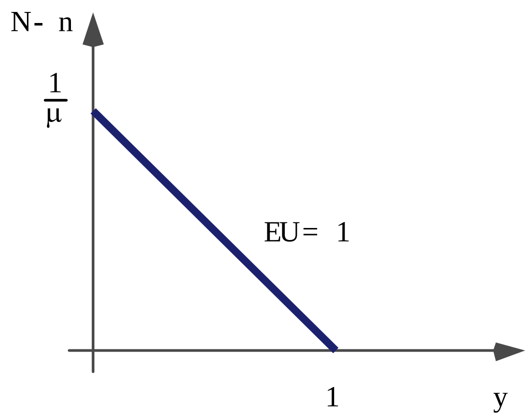


Symmetric Information

Suppose owners do not know quality either. Then the information is symmetric. Group one's problem is in (14) and the expected utility is

$$E\left\{\int_n^N X(t)\ dt\right\} = \mu\left(N - n\right)$$

A representative indifference curve is shown in the following figure.



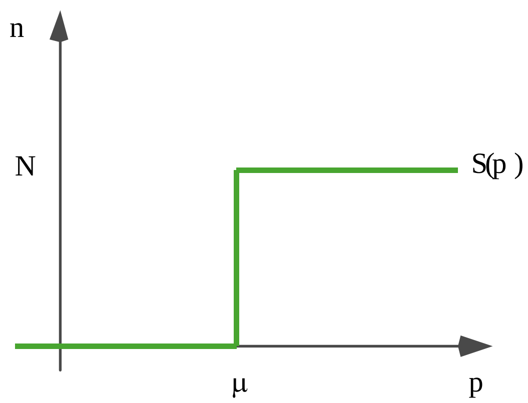
The marginal rate of substitution is μ^{-1} while the terms of trade are indicated by p^{-1}. Hence

$$N - n = \begin{cases} 0 & p > \mu \\ \left[0, N\right] & p = \mu \\ N & p < \mu \end{cases}$$

and so

$$n = \begin{cases} N & p > \mu \\ \left[0, N\right] & p = \mu \\ 0 & p < \mu \end{cases}$$

This supply function is shown in the next figure.



The demand does not change in this symmetric information case and so there are three possible equilibria; the cases correspond to (i) m < N, (ii) $\frac{2}{3}m < N < m$, and (iii) $N < \frac{2}{3}m$.

Figure: Equilibria given Asymmetric Information