I. Introduction

Investment has long been considered present sacrifice for future benefit, i.e., most notably by Irving Fisher.[1] Hirshleifer notes that

> ". . . the present is relatively well known, whereas the future is always an enigma. Investment is also, therefore, *certain* sacrifice for *uncertain* benefit."[2]

In the theory of investment behavior, Fisher's work remains a classic. Although Fisher's theory explains much of the observed investment behavior, it does not provide an explanation for different returns or yields on various forms of investment. We will construct a model which retains the main features of the Fisher model and incorporates uncertainty. The introduction of uncertainty will add enough explanatory power to the model to deal with the existence of different returns or yields on the assets. In addition the model should add enough explanatory power for us to not only value corporate equity but also various kinds of debt instruments.

In section II, the simplest version of the Fisher model under uncertainty is constructed. It is a two date model in which investors make savings and portfolio decisions, and firms make production, investment, and financing decisions. In section III, the objective function of the publicly held and traded corporation is derived. In the first case considered, the manager is given a compensation package which includes a known salary now and then and an initial stock ownership position in the corporation. In this case we show that the Fisher Separation result holds in this complete financial market model and that it is also a unanimity result, i.e., all current shareholders agree with the investment decision made by the corporate manager. In the second case considered, the manager is given a compensation package which includes a known salary now and then and a number of stock options. In this case we show that the

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[1]Irving Fisher, *The Theory of Interest*, New York, MacMillan, 1930.

[2]See Hirshleifer's "Investment Decision Under Uncertainty: Choice-Theoretic Approaches," *The Quarterly Journal of Economics*, LXXXIX (November 1965), p. 509.

manager makes decisions on corporate account to maximize the value of the stock option rather than the stock market value.

II. The Fisher Model under Uncertainty

Consider a competitive economy operating between the dates *now* and *then*. The consumer i selects a consumption pair (c_{i0}, c_{i1}) where c_{i0} denotes the dollar value of consumption now and c_{i1} denotes the dollar value of consumption then. Let (m_{i0}, m_{i1}) denote the consumer's income now and then. Let $u_i(c_{i0},c_{i1})$ be the consumer's increasing concave utility function; u_i expresses the individual's preferences for consumption now versus then. In order to introduce uncertainty let $\Omega = \{\omega_1, \omega_2, \omega_3, \ldots, \omega_n\}$ denote the set of states of nature thant can occur then. Each state ω is regarded as an index of economic conditions and the states are ordered so that larger ω correspond to better economic conditions. Finally, let $\phi_i(\omega)$ denote the probability of state ω, i.e., agent i's assessment of the probability that state ω will occur then.

To make the uncertainty operational suppose that the consumer can only transfer dollars from now to then by purchasing one or more of the n risky basis assets; each basis asset, in a **complete financial market** model, is a promise to pay one dollar if state of nature ω occurs and zero otherwise. Let $p(\omega)$ be the price of a basis asset of type ω.

Consumer/Investor Behavior

The consumer selects a consumption plan which specifies a consumption level now and a consumption level then for each state of nature. In its classic form, the problem is stated as a constrained maximization problem. Due to the uncertainty concerning consumption then, the agent maximizes expected utility subject to the budget constraint. If there are two states of nature then the agent selects $(c_{i0}, c_{i1}(\omega_1), c_{i1}(\omega_2))$ to

$$\text{maximize } u_i((c_{i0}, c_{i1}(\omega_1))\ \phi_i(\omega_1) + u_i((c_{i0}, c_{i1}(\omega_2))\ \phi_i(\omega_2)$$

$$\text{subject to } c_{i0} + p(\omega_1)\ c_{i1}(\omega_1) + p(\omega_2)\ c_{i1}(\omega_2) = m_{i0} + p(\omega_1)\ m_{i1}(\omega_1) + p(\omega_2)\ m_{i1}(\omega_2)$$

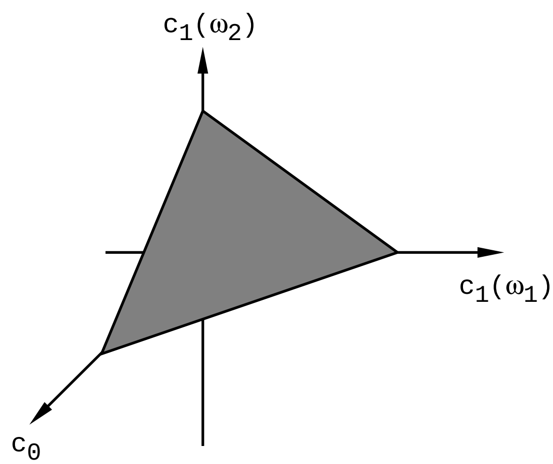
In the general case, the problem is

$$\text{maximize } \sum_\Omega u_i(c_{i0}, c_{i1}(\omega))\ \phi_i(\omega)$$

$$\text{subject to } c_{i0} + \sum_\Omega p(\omega)\ c_{i1}(\omega) = m_{i0} + \sum_\Omega p(\omega)\ m_{i1}(\omega),$$

The left hand side of the constraint is the risk adjusted present value of consumption and the right hand side is the risk adjusted present value of income. The budget constraint, appropriate for the n = 2 case, is shown in the following figure.

Figure 1: the budget constraint



For the case n = 2, it should be clear that the individual selects a position on the budget plane to which none is preferred. The individual's preferences are represented by expected utility and in this three dimensional space those preferences may be represented by indifference surfaces. Then, the indifference surface which is tangent to the budget plane locates the optimal consumption triple.

Consider the following method for locating the optimal consumption plan. Suppose the consumer moves along the budget plane and notes the value of expected utility with each movement. In particular note that the level surfaces of the expected utility function, which we call indifference surfaces, intersect the budget plane in curves along which expected utility remains constant. To find a maximum for the expected utility function on the budget plane, the consumer moves across the indifference surfaces in a direction which increases expected utility. When the consumer locates a maximum, there is no direction in the budget plane in which expected utility increases. The consumption bundle which has an indifference surface tangent to the budget plane satisfies this condition.

To further characterize the conditions for an optimal consumption plan consider two dimensional slices of the budget plane and indifference surface. If the consumption then in state two is held fixed at $c_{i1}^{0}(\omega_2)$ then the budget constraint can be written as

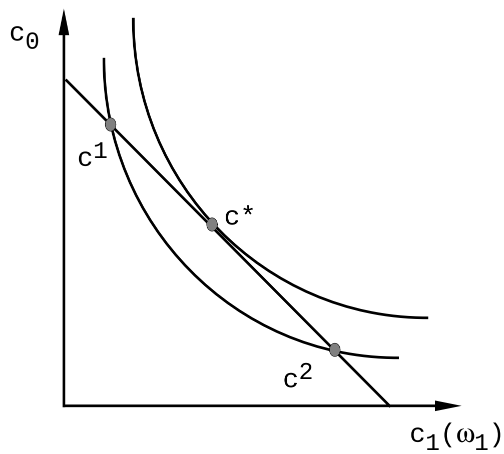
$$c_{i0} + p(\omega_1)\, c_{i1}(\omega_1) = m_{i0} + p(\omega_1)\, m_{i1}(\omega_1) + p(\omega_2)\, (m_{i1}(\omega_2) - c_{i1}^{0}(\omega_2)),$$

or equivalently, as

$$c_{i0} = \left[m_{i0} + p(\omega_1)\, m_{i1}(\omega_1) + p(\omega_2)\, (m_{i1}(\omega_2) - c_{i1}^{0}(\omega_2))\right] - p(\omega_1)\, c_{i1}(\omega_1)$$

Note that the intercept is the risk adjusted present value of the individual's income minus the risk adjusted present value of the consumption planned in state two. Also, observe that the slope of this slice of the budget constraint is - $p(\omega_1)$. This slice of the budget constraint is shown in figure two. The price now of another dollar of consumption then in state ω_1 is $p(\omega_1)$. Equivalently, $p(\omega_1)$ is the cost of another dollar then in state $\omega_{1,}$ in terms of dollars now.

figure 2: the safe versus risky consumption choice



The indifference curves represent the consumption pairs, i.e., then and now, which yield the same expected utility. The steepness of an indifference curve at a particular consumption pair is the marginal rate of substitution. At the consumption pair c^1 in figure two, the marginal rate of substitution is greater than the opportunity cost, i.e.,

$$mrs_{10}(c^1) > p(\omega_1).$$

The marginal rate of substitution expresses the individual's temporal value assessment at the margin c^1. Equivalently, **the marginal rate of substitution expresses the value of dollars then in state one, in terms of dollars now**. **The inequality can be interpreted as saying that the individual values an additional dollar then in state one, at more than its cost**. Hence, trading $p(\omega_1)$ dollars of consumption now for another dollar of consumption then in state one increases the individual's expected utility now. At the consumption pair c^2 in figure two,

$$mrs_{10}(c^2) < p(\omega_1).$$

This inequality can be interpreted as saying that the individual values an additional dollar then in state one, at less than its cost. Hence, trading a dollar of consumption then in state one for another $p(\omega_1)$ dollars now increases the individual's expected utility now. It follows that, other things being equal, the individual's condition for an optimal consumption pair is

$$mrs_{10}(c^*) = p(\omega_1),$$

as shown in the figure.

To further characterize the conditions for an optimal consumption plan consider another two dimensional slice of the budget plane and indifference surface. If the consumption now is held fixed at c^0_{i0} then the budget constraint can be written as

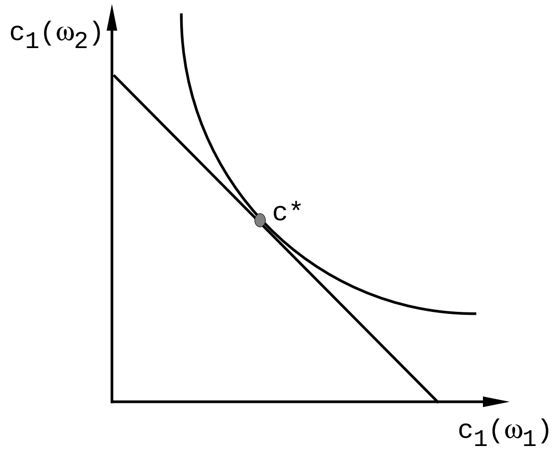
$$p(\omega_1)\ c_{i1}(\omega_1) + p(\omega_2)\ c_{i1}(\omega_2) = m_{i0} - c^0_{i0} + \sum_\Omega p(\omega)\ m_{i1}(\omega),$$

or equivalently, as

$$c_{i1}(\omega_2) = \frac{m_{i0} - c^0_{i0} + \sum_\Omega p(\omega) m_{i1}(\omega)}{p(\omega_2)} - \frac{p(\omega_1)}{p(\omega_2)}\, c_{i1}(\omega_1)$$

Note that the intercept is the risk adjusted future value of the individual's income minus the risk adjusted future value of the fixed consumption now. Also, observe that the slope of this slice of the budget constraint is $- p(\omega_1)/p(\omega_2)$. Note that this price ratio has a standard interpretation, i.e., the price ratio represents the cost of state one dollars in terms of state two dollars. This slice of the budget constraint is shown in the figure three.

figure 3: the risky versus risky consumption choice



The indifference curves represent the consumption pairs, i.e., consumption in states one and two, which yield the same expected utility. The steepness of an indifference curve at a particular consumption pair is the marginal rate of substitution. In this case the marginal rate of substitution reflects the value of state one dollars in terms of state two dollars. Other things being equal, the optimal pair is determined by the condition

$$mrs_{12} = \frac{p(\omega_1)}{p(\omega_2)}$$

which says that at the margin the value of state one dollars in terms of state two dollars is equal to the cost of state one dollars in terms of state two dollars.

In each statement of the problem, the individual has an incentive to hold more than one type of asset, in spite of the fact that the prices differ.[3] This, of course, is due to risk aversion, i.e., the risk averse individual prefers a payoff in both states to a payoff in one, even

[3] It need not be true that $x_i(\omega) \geq 0 \; \forall \; \omega \in \Omega$. However, even if $x_i(\omega) \geq 0$ it does not follow that the consumer is not selling shares in some corporation short because the corporations are simply portfolios of the stocks modeled here.

if the prices of the assets differ.[4] Hence this model provides an explanation for the existence of different rates of return; recall that the rate of return on an asset which has a one dollar payoff in state ζ is

$$r(\zeta) = \frac{1}{p(\zeta)} - 1$$

and so if $p(\zeta) > p(\xi)$ it follows that $r(\zeta) < r(\xi)$.

Corporate Payoffs and Stock Market Values

Consider a publicly held and traded corporation in this complete financial market economy. Suppose that corporate earnings are random, i.e., depend on the index of economic activity. Let $\Pi_f(\omega)$ denote the payoff of corporation f then. Suppose the corporation has previously issued N_f shares of common stock. If the firm is unlevered then the payoff per share of common stock is Π_f/N_f. Recall that the price of a basis asset of type ζ is $p(\zeta)$ and so the value now of a payoff of $\Pi_f(\zeta)/N_f$ dollars then in state ζ is

$$p(\zeta)\,\frac{\Pi_f(\zeta)}{N_f}$$

An equivalent statement can be made for each state.[5] Hence, the stock value of the corporation is the risk adjusted present value of its payoff, or equivalently, it quasi-rent. Letting p_f denote the common stock price of corporation f, it follows that

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[4]It would be possible, at this point, to derive a number of comparative static results, e.g., consider how the individual's family of indifference curves for the expected utility function change when the state probabilities $\phi_i(\omega)$ change or when the individual becomes more risk averse.

[5]Also, recall that the price of the basis asset ζ is

$$p(\zeta) = \frac{1}{1+r(\zeta)}$$

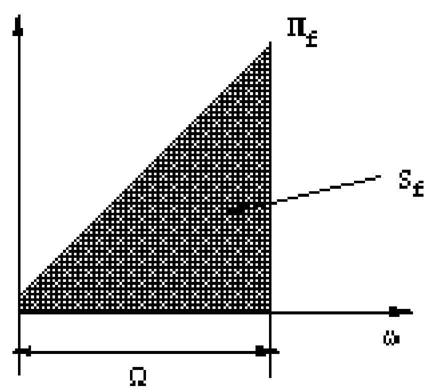
$$p_f = \sum_{\Omega} p(\omega) \frac{\Pi_f(\omega)}{N_f}. \quad (1)$$

Similarly, letting S_f denote the stock market value of corporation f, it follows that

$$S_f \equiv p_f N_f = \sum_{\Omega} p(\omega)\, \Pi_f(\omega). \quad (2)$$

For this representation it is assumed that the corporation does not have any debt. The stock value may be interpreted as the risk adjusted present value of the area shown in the following figure.[6]

figure 4: stock market value



where $r(\zeta)$ is a risk adjusted interest rate. It follows that

$$p(\zeta) \frac{\Pi_f(\zeta)}{N_f} = \frac{1}{1+r(\zeta)} \frac{\Pi_f(\zeta)}{N_f}$$

i.e., the value now of the payoff is a risk adjusted present value.

[6]To justify this description, note that by the Mean Value Theorem of the Integral Calculus there exists a state $\zeta \in \Omega$ such that

$$\int_{\Omega} p(\omega)\, \Pi_f(\omega)\, d\omega = p(\zeta) \int_{\Omega} \Pi_f(\omega)\, d\omega.$$

The last integral represents the shaded area and $p(\zeta)$ represents the risk adjusted discount factor.

The share price of the corporation's common stock can also be developed using a no arbitrage argument. To see this, suppose the corporation's share price is less than the risk adjusted present value of the payoff per share, i.e., by hypothesis

$$p_f < \sum_{\Omega} p(\omega) \frac{\Pi_f(\omega)}{N_f} \tag{H}$$

If the hypothesis (H) holds, then we want to show that an arbitrage opportunity exists. Consider the portfolio an investor can form to take advantage of (H). Suppose the investor purchases one share of corporate stock at a price p_f. Also, suppose this investor finances this purchase of corporate stock by going short in a portfolio of basis stock. If $\Pi_f(\omega_1)/N_f$ shares of basis stock of type one are sold short, then the investor obtains $p(\omega_1)\ \Pi_f(\omega_1)/N_f$ dollars now. Hence, suppose the investor the following portfolio of short positions:

$$\left(\frac{\Pi_f(\omega_1)}{N_f}, \frac{\Pi_f(\omega_2)}{N_f}, \ldots, \frac{\Pi_f(\omega_n)}{N_f} \right) \tag{P}$$

The risk adjusted net present value of these transactions is

$$npv = -\ p_f + \sum_{\Omega} p(\omega) \frac{\Pi_f(\omega)}{N_f} > 0$$

by (H). Also, observe that this portfolio obligates the investor to pay $\Pi_f(\omega)/N_f$ dollars then but that same amount is paid on the share of corporate stock which was purchased. Hence, the investor increases the risk adjusted present value of his income without any additional risk then. Since (H) is an arbitrage condition, it follows that the corporation's share price cannot fall below the value of the basis stock portfolio specified in (P).

It is possible to consider new financing in the context of this model. Suppose corporation f invests I_f dollars in a project which yields a random payoff of $\Pi_f(I_f, \omega)$ dollars. Suppose the corporate manager of the publicly held and traded corporation f makes the investment decision for the firm now and uses a new stock issue to finance the investment. Let S_f^n denote the value of the new stock issue. Suppose the firm has issued N_f shares of stock previously and issues n_f new shares to finance the investment of I_f dollars. Note that the value of the new issue is

$$S_f^n = \sum_{\Omega} p(\omega) \frac{n_f}{N_f+n_f} \Pi_f(I_f, \omega) = \frac{n_f}{N_f+n_f} S_f, \tag{3}$$

where S_f is the stock market value of the new and old issues. The old stockholders have a fractional ownership of

$$1 - \frac{n_f}{N_f+n_f} = \frac{N_f}{N_f+n_f}$$

and so the stock market value of the old shareholders' position in the firm is S_f^o, where

$$S_f^o = \frac{N_f}{N_f+n_f} S_f. \tag{4}$$

Now, suppose the manager issues enough new shares to cover just the investment expenditure of I_f dollars, i.e., $S_f^n = I_f$. It follows that the firm will have to sell enough new shares so that the new shareholders' proportional ownership of the firm is

$$\frac{n_f}{N_f+n_f} = \frac{I_f}{S_f}$$

Similarly, the old shareholders' equity stake in the corporation is found by noting that

$$S_f^o = S_f - S_f^n$$

$$\Leftrightarrow \; S_f^o = S_f - I_f$$

$$\Leftrightarrow \frac{N_f}{N_f + n_f} = 1 - \frac{I_f}{S_f} = \frac{S_f - I_f}{S_f}$$

Since $S_f - I_f$ is the risk adjusted net present value of the corporation, it should be noted that the old shareholders' equity stake is larger for projects with larger risk adjusted net present values.

exercises:

[1] Explain why $S_f \geq I_f$ is a necessary condition for investing I_f dollars now.

[2] Reverse the inequality in (H). Develop an arbitrage argument to show that the converse of (H) cannot hold either.

[3] Suppose that corporation f consists of two projects with quasi-rents $\Pi_1(\omega)$ and $\Pi_2(\omega)$, i.e., $\Pi_f = \Pi_1 + \Pi_2$. Derive an expression for the stock market value of the corporation.

[4] Suppose the corporation can select one of two mutually exclusive projects with random payoffs Π_1 and Π_2. Suppose the investment expenditure is the same for both projects and that $\Pi_1 \leq \Pi_2$ for all $\omega \in \Omega$. Which project will be selected?

[5] Define a safe bond contract as a promise to pay one dollar then, independent of which state occurs. Then a bond issue with a promised repayment of B_f dollars then represents B_f bond contracts. Show that the price of this bond contract is

$$\delta \equiv \sum_{\Omega} p(\omega)$$

and that the value of the bond issue is D_f, where

$$D_f = \sum_{\Omega} p(\omega)\, B_f.$$

Provide a graphical interpretation.

[6] Define a risky bond contract as a promise to pay one dollar then if the corporate payoff is sufficient to cover the promised payment of B_f dollars. If the corporate payoff is not sufficient to cover the promised payment, let $\Pi_f(\omega)/B_f$ denote the payoff per bond contract. Show that the payoff on the bond issue is $\min\{\Pi_f(\omega), B_f\}$ and provide an expression for the bond market value, i.e., D_f, of the corporate bond issue. Provide a graphical interpretation.

[7] Let V_f^u and V_f^l denote the corporate value of an unlevered and levered corporation (i.e., a firm with a bond issue), respectively. Corporate value is defined as the value of the firm's financial contracts, i.e., $V_f^u \equiv S_f$ and $V_f^l \equiv D_f + S_f^l$. Using [5], show that $V_f^u = V_f^l$..

III. Executive Compensation and Fisher Separation

Now, consider the manager of a publicly held corporation. Suppose the manager makes the investment decision for the firm now and uses a new stock issue to finance the investment, i.e., $S_f^n = I_f$. The current shareholders of the corporation want the manager to act in their interests, i.e., maximize S_f^o, and the manager has a fiduciary responsibility to act in the stockholders' interests. Here, however, we allow the manager to pursue his or her own self interest. Two cases are considered. In each case, the manager has a known salary now and then. In the first case, the manager is a current shareholder of the corporation. In the second case, the manager is not a current shareholder but is provided with a stock option package.

Case I: Stock

Let manager i of corporation f have x_{if}^0 shares of corporate stock now, where $x_{if}^0 > 0$. The manager can trade the basis and corporate stock on personal account. Let x_{if} denote the number of shares of corporate stock held by the manager after trading, i.e., $x_{if}^0 - x_{if}$ is the change in the number of shares of corporate stock. Let p_f denote the share price of the corporate stock. Then, note that

$$S_f \equiv p_f\,(N_f + n_f) = \sum_\Omega p(\omega)\,\Pi_f(I_f, \omega). \tag{5}$$

Equivalently,

$$p_f = \frac{1}{N_f+n_f} \sum_\Omega p(\omega)\,\Pi_f(I_f, \omega). \tag{6}$$

Also suppose that the manager has an income pair (m_{i0}, m_{i1}). This income pair may be interpreted as part of the manager's compensation scheme. The manager, acting in his self-interest, selects the investment level I_f on corporate account and a stock portfolio on personal account to maximize expected utility subject to the budget constraint, i.e.,

$$\text{maximize} \sum_\Omega u_i(c_{i0}, c_{i1}(\omega))\,\phi_i(\omega)$$
$$\text{subject to } c_{i0} + \sum_\Omega p(\omega)\,c_{i1}(\omega) = m_{i0} + p_f\,(x_{if}^0 - x_{if}) + \sum_\Omega p(\omega)\,m_{i1} + \sum_\Omega p(\omega)\,x_{if}\,\frac{\Pi_f(I_f,\omega)}{N_f+n_f}$$

Observe that

$$p_f\,(x_{if}^0 - x_{if}) + \sum_\Omega p(\omega)\,x_{if}\,\frac{\Pi_f(I_f,\omega)}{N_f+n_f}$$

represents the risk adjusted present value of the manager's stake in the corporation and this stake depends on the investment decision made on corporate account. It is somewhat less apparent that it does not depend on any change in the manager's equity position. Using (4)-(6), note that the manager's stake can be rewritten as

$$\frac{x_{if}^0 - x_{if}}{N_f + n_f} S_f + \frac{x_{if}}{N_f + n_f} S_f =$$

$$\frac{x_{if}^0}{N_f + n_f} S_f =$$

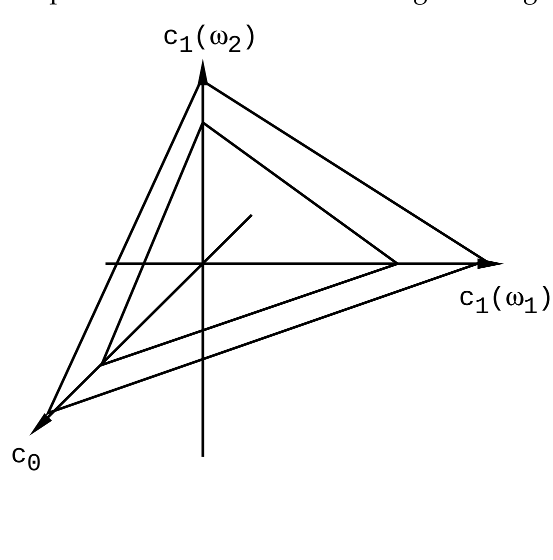
$$\frac{x_{if}^0}{N_f} S_f^o$$

Therefore, the manager's decision problem may be equivalently expressed as

$$\text{maximize } \sum\nolimits_\Omega u_i(c_{i0}, c_{i1}(\omega))\, \phi_i(\omega)$$
$$\text{subject to } c_{i0} + \sum\nolimits_\Omega p(\omega)\, c_{i1}(\omega) = m_{i0} + \frac{x_{if}^0}{N_f} S_f^o + \sum\nolimits_\Omega p(\omega)\, m_{i1}$$

In this form, it is clear that the manager makes decisions on corporate account which maximize the stock market value of the current shareholders. Since $S_f^o = S_f - S_f^n = S_f - I_f$, the manager selects the investment level on corporate account to maximize S_f^o, or equivalently, to maximize $S_f - I_f$. Maximizing this risk adjusted net present value increases the risk adjusted present value of the manager's income stream.

figure 5: corporate decisions and the manager's budget constraint



Note that, in either form, the investment decision does not depend on the manager's risk preferences or probability beliefs. Equivalently, the Fisher Separation result goes through for the publicly held and traded corporation, in the case of an equity issue. Hence, the manager, who owns stock in the corporation now, has the incentive to act in the interests of all shareholders. This incentive translates into a maximization of the risk adjusted present value of the corporation.

Case II: Stock Options

Next, consider the value of a stock option. A stock option is simply a warrant. Suppose the warrant gives its owner the right to purchase one share of common stock, at an exercise price of e_f dollars per share. Suppose corporation f has N_f shares of common stock outstanding and has issued n_f warrants to its manager. Also recall that in order to finance the investment a new equity issue of m_f is necessary. Of course, the number m_f of new shares is determined by the size of the investment. Then the firm must issue n_f new shares if the warrant is exercised. Let $\gamma = n_f/(N_f + m_f + n_f)$. Then γ is fraction of the corporate payoff obtained by exercising the warrant. The payoff on the warrant issue is

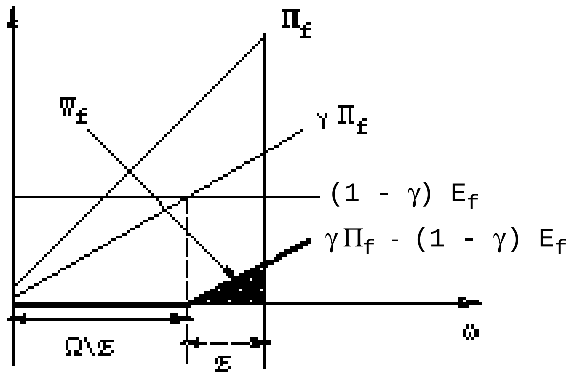
$$\max\left\{0, n_f\left(\frac{\Pi_f + E_f}{N_f + m_f + n_f} - e_f\right)\right\} = \max\left\{0, \gamma\left(\Pi_f + E_f\right) - E_f\right\},$$

where $E_f \equiv e_f\, n_f$ is the exercise value of the warrant issue. Then it follows easily that the market value of the warrant is

$$W_f = \sum\nolimits_\Omega p(\omega)\max\left\{0, \gamma\left(\Pi_f + E_f\right) - E_f\right\} = \sum\nolimits_\Omega p(\omega)\max\left\{0, \gamma\,\Pi_f - \left(1-\gamma\right)E_f\right\}.$$

The risk adjusted present value of the shaded area in the next figure represents the warrant value. Let w_f denote the warrant price now.

figure 6: warrant payoff and value



Then

$$w_f = \frac{W_f}{n_f}.$$

Note that the price of this warrant can also be expressed as

$$w_f = \frac{1}{n_f} \sum_{\Omega} p(\omega) \max\left\{0, \gamma\,(\Pi_f + E_f) - E_f\right\}$$

$$= \frac{1}{n_f} \sum_{E} p(\omega) \left[\frac{n_f}{N_f+m_f+n_f} (\Pi_f + E_f) - e_f\, n_f\right]$$

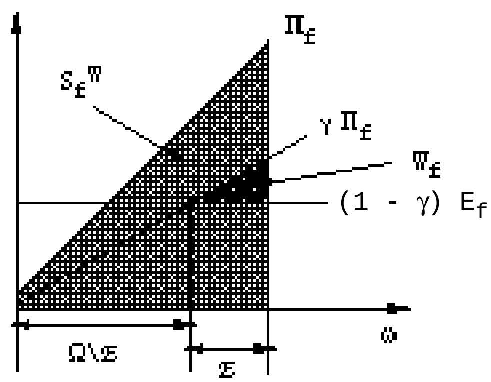
$$= \sum_{E} p(\omega) \left[\frac{\Pi_f + E_f}{N_f + m_f + n_f} - e_f\right]$$

The stockholders in this corporation have a payoff Π_f in the event that the options are not exercised and $\Pi_f - \max\{0, \gamma(\Pi_f + E_f) - E_f\}$ in the event that the options are exercised. Hence, letting S_f^W denote the stock market value of the firm with a warrant issue, we obtain

$$S_f^w = \sum_\Omega p(\omega)\,\Pi_f(\omega) - \sum_\Omega p(\omega)\max\{0, \gamma\left(\Pi_f(\omega) + E_f\right) - E_f\} = S_f - W_f,$$

i.e., the stock market value of the firm with a warrant issue is equivalent to the stock market value of the firm minus the value of the warrant issue. With the usual qualification, these values are represented in the following figure.

figure 7: stock and warrant values



Now, consider the behavior of a corporate manager with a compensation scheme which includes a stock option but no initial shares in the corporation. The manager may sell the warrants now or hold them and possibly exercise them then. If the manager sells the warrants now, he receives $w_f\, n_f$ dollars. Let y_{if} denote the number of warrants the manager has after trading in the financial market. Then $w_f\,(n_f - y_{if})$ is the income generated now. Let E denote the event that the warrant, equivalently, call option, is in the money then and let $\Omega\backslash E$ denote the event that the warrant is out of the money then. Note that the event that the warrant is in the money is the exercise event and it is defined as

$$E = \left\{ \omega \in \Omega \mid \frac{\Pi_f(I_f, \omega) + E_f}{N_f + m_f + n_f} > e_f \right\}$$

If the manager holds some of the warrants, his warrant payoff then is

$$= \begin{cases} 0 & \omega \in \Omega \setminus E \\ y_{if} \dfrac{\Pi_f(I_f, \omega) + E_f}{N_f + m_f + n_f} - e_f y_{if} & \omega \in E \end{cases}$$

It follows that the manager's budget constraint in the financial market is

$$c_{i0} + \sum\nolimits_{\Omega} p(\omega)\, c_{i1}(\omega) = m_{i0} + w_f\,(n_f - y_{if}) + \sum\nolimits_{\Omega} p(\omega)\, m_{i1}(\omega)$$

$$+ \sum\nolimits_{E} p(\omega) \left(y_{if} \frac{\Pi_f(I_f,\omega)+E_f}{N_f+m_f+n_f} - e_f\, y_{if} \right)$$

Finally, observe that any trading the manager does in warrants yields a zero risk adjusted net present value since

$$w_f\,(n_f - y_{if}) + y_{if} \sum\nolimits_{E} p(\omega) \left(\frac{\Pi_f(I_f,\omega)+E_f}{N_f+m_f+n_f} - e_f \right) =$$

$$w_f\, n_f - w_f\, y_{if} + w_f\, y_{if} =$$

$$w_f\, n_f.$$

The risk adjusted net present value of trading in the warrants is

$$npv = -\, w_f\, y_{if} + y_{if} \sum\nolimits_{E} p(\omega) \left(\frac{\Pi_f(I_f,\omega)+E_f}{N_f+m_f+n_f} - e_f \right) =$$

$$- w_f \, y_{if} + w_f \, y_{if} = 0$$

It follows that the manager's budget constraint may be equivalently expressed as

$$c_{i0} + \sum_{\Omega} p(\omega) \, c_{i1}(\omega) = m_{i0} + W_f + \sum_{\Omega} p(\omega) \, m_{i1}(\omega).$$

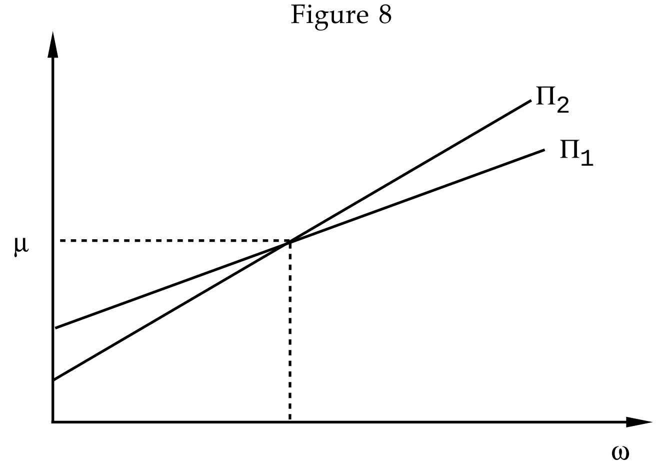
Hence, the corporate manager makes decisions on corporate account which maximize the value of the warrant.

exercises:

[8] In case II, show that if the exercise price of the stock options is zero, i.e., $e_f \equiv 0$, then the manager selects the investment level to maximize current shareholder value, i.e., $S_f^o(I_f)$. Explain.

[9] Suppose the manager has the stock option compensation scheme. Also, suppose the manager can select one of the two mutually exclusive projects, i.e., Π_1 and Π_2, shown in the following figure. Finally, suppose that the investment expenditure is the same for each project.

Figure 8



Will project selection depend on the magnitude of the exercise price? Explain. Provide a graphical interpretation.